Close Friday: HW_6A, 6B, 6C (7.4,7.5,7.7)
See the many practice problems and summary sheets I posted online.

7.4 Partial Fractions (continued)

Aside: Here are examples of **all** the types of integrals you will see in section 7.4.

$$\int \frac{1}{2x+5}\,dx = \frac{1}{2}\ln|2x+5| + C$$

$$\int \frac{1}{(x-4)^2}\,dx = -\frac{1}{x-4} + C$$

$$\int \frac{1}{(x+7)^3}\,dx = -\frac{1}{2}\frac{1}{(x+7)^2} + C$$

$$\int \frac{1}{x^2+9}\,dx = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\int \frac{x}{x^2+9}\,dx \quad (\text{use } u = x^2+9)$$

Entry Task: Integrate

$$\int \frac{2x+3}{x^3+6x^2+9x}\,dx = \int \frac{2x+3}{x(x+3)^2}\,dx$$

Step 1: Factor (I did this for you above).

Step 2: Write

$$\frac{2x+3}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

Do this now:

Find A, B, and C like you did in the worksheet!

Step 3: Integrate

Summary of Partial Fractions

Given a *rational* function $\frac{p(x)}{q(x)}$.

0. If the highest power of p(x) is bigger than q(x), divide and simplify.

1. Factor the denominator, q(x).

2. Write the decomposition:

i) Distinct Linear Factors:

$$\frac{x^2-3}{x(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4}$$

ii) Repeated Linear Factors:

$$\frac{5+2x}{(x+3)(x-2)^3} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

iii) Irreducible Quadratic Factors:

$$\frac{4x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

3. Solve for A, B, C …. Then integrate!

Example of last case:

$$\int \frac{x^2-x+6}{x^3+3x}\,dx = \int \frac{x^2-x+6}{x(x^2+3)}\,dx$$

How to integrate

1. Look for simplifications/substitutions
2. Products/Logs/Inverse Trig → BY PARTS
   Sin/Cos/Tan/Sec combos → TRIG
   Quadratic (under a radical) → TRIG SUB
   Rational Function → PART. FRAC.
3. If nothing seems to work, substitution.
   (u = inside, u = $\sqrt{\ }$, u = trig, u = e^x)

Here are four problems that don't obviously fit any of our four special methods, which means they all start with substitution!

1. $\int e^{\sqrt{x}} dx$

2. $\int \frac{3}{x - 2\sqrt{x}} dx$

3. $\int \frac{\cos(x)}{4 - \sin^2(x)} dx$

4. $\int e^x \cos(e^x) \sin^3(e^x)\, dx$

How would you start these:

1. $\int \tan^3(x) \sec(x)\, dx$

2. $\int x^2 \ln(x)\, dx$

3 $\int x \sqrt{5 - x^2} dx$

4. $\int \frac{\sqrt{x^2 - 1}}{x^2} dx$

5. $\int \frac{x^2 + 1}{x^2 - 2x - 3} dx$

6. $\int x \tan^{-1}(x) dx$

7. $\int \frac{dx}{\sqrt{4x^2 + 8x - 12}} dx$

7.7 Approximating Integrals: Despite our best efforts in 7.1-7.5, the vast majority of integrals can NOT be done with any of our methods.

To approximate $\int_a^b f(x)dx$

1. Pick **n = number of subdivisions**.
   Compute $\Delta x = \frac{b-a}{n}$.
2. Label the tick marks: $x_i = a + i\Delta x$
3. Use an approximation method:

$$L_n = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_{n-1})] \quad \text{(Left endpoint)}$$
$$R_n = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_n)] \quad \text{(Right endpoint)}$$
$$M_n = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \quad \text{(Midpoint)}$$

Trapezoid Rule: (all the "middle terms" are multiplied by 2)

$$T_n = \frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Simpson's Rule: *n* must be even! (Alternating multiplying middle terms by 4 and 2)

$$S_n = \frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$\int_0^3 \sqrt{100 - x^3}\,dx$$

$$L_3 = (1)\left[\sqrt{100-(0)^3} + \sqrt{100-(1)^3} + \sqrt{100-(2)^3}\right] \approx 29.5415$$

$$R_3 = (1)\left[\sqrt{100-(1)^3} + \sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.0855$$

$$M_3 = (1)\left[\sqrt{100-(0.5)^3} + \sqrt{100-(1.5)^3} + \sqrt{100-(2.5)^3}\right] \approx 29.0091$$

$$T_3 = \frac{1}{2}(1)\left[\sqrt{100-(0)^3} + 2\sqrt{100-(1)^3} + 2\sqrt{100-(2)^3} + \sqrt{100-(3)^3}\right] \approx 28.8135$$

$$S_6 = \frac{1}{3}\cdot\frac{1}{2}\Big[\sqrt{100-(0)^3} + 4\sqrt{100-(0.5)^3} + 2\sqrt{100-(1)^3} + 4\sqrt{100-(1.5)^3}$$
$$+2\sqrt{100-(2)^3} + 4\sqrt{100-(2.5)^3} + \sqrt{100-(3)^3}\Big] \approx 28.9441$$

"Actual" (to 8 places after decimal): 28.94418784